Abstract—Shunt FACTS devices provide the possibility to control voltages and therefore to improve the security of the system. In order to use this possibility, the optimal location and the set values of the FACTS controllers have to be selected correctly. The singular analyses of the power system Jacobian matrix are applied to identify the optimal location of shunt FACTS devices in large power systems. Furthermore the application of Optimal Power Flow control is a possible method for choosing set values. In this paper a sensitivity index for the detection of sensor nodes is defined. Sensitivity analysis is used to determine the area on which the FACTS device has significant influence. And then only this limited area is included in the Optimal Power Flow control, because it is very difficult to include the entire system into the optimization process. Simulations are performed on an IEEE 57-bus system. Furthermore the objective function of Optimal Power Flow control is analyzed. This function consists of three components. A certain type of FACTS is not able to influence all these components. In this paper it is evaluated for which component the shunt FACTS devices are mainly responsible.

I. INTRODUCTION

Corresponding to the increasing capacity utilization of power systems due to increased offshore wind potential, new strategies as load flow and voltage control can increase the available transfer capacities and improve voltage profiles. The fast development of power electronics enable the development of different devices used in AC systems. These devices are often referred to as FACTS (Flexible AC Transmission Systems)[1]. FACTS devices are able to influence power flows and voltages to different degrees depending on the type of the device [2]. The focus in this paper lies on the Static Var Compensator SVC. SVC can be integrated into the system for voltage control at the weak and sensor nodes. Therefore the identification of weak and sensor nodes plays a major role for the optimal location of shunt FACTS devices [1].

Eigenvalue analysis of network Jacobian matrix is a suitable methodology to seek the optimal location of SVC in a power system. The magnitudes of the eigenvalues provide a relative measure of proximity to instability. On the other hand the eigenvectors provide information related to the mechanism of loss of voltage stability [3]. With help of calculated eigenvalues and eigenvectors the sensor and weak nodes can be eliminated.

Furthermore in order to profit from the advantages of shunt FACTS, suitable set values for these devices have to be determined. A useful option is the application of Optimal Power Flow control. The set values are determined such that an objective function is minimized given the system model. For the definition of the objective function, voltage deviations from given reference level, line loading and active power losses are taken into account. The influence of FACTS devices are not confined to one bus or line, but the area on which the devices have a considerable influence is limited. Therefore only this influence area is considered for the optimization process. This allows a reduced problem size and a fast optimization process.

This paper is organized as follows: Following the introduction, the generalities on FACTS devices and the operating mode of the static var compensator (SVC) to voltage controlling are described in section II. Then in section III the methodology to detect of sensor and weak nodes by the analysis of Jacobian matrix is introduced and a sensitivity index for the detection is proposed. Section IV describes the determination of the influence area by sensitivity analysis. In section V the optimal Power Flow control problem is introduced. The structure of a test system for the analysis and the analysis of the objective function are presented in section VI. Finally, brief conclusions are deduced.

II. FACTS DEVICES

In a power system, the FACTS devices may be applied to achieve several aims: to operate transmission lines close to their thermal limits and to reduce the loop flows by supplying or absorbing reactive power, increasing or reducing of voltage and controlling series impedance or phase angle. Accordingly, three categories of FACTS devices can be distinguished [4], [5]:

- series devices,
- shunt devices,
- combined devices.

Fig. 1 shows the power flow control and stability improvement in a power system using FACTS devices. The active power transmitted between the systems 1 and 2 is
defined by the equation shown in Fig. 1. Thereby \( U_1 \) and \( U_2 \) are the voltages at both ends of the transmission line, \( X \) is the impedance of the line and \((\theta_1 - \theta_2)\) is the phase angle difference between both systems. It can be seen, that the transmitted power can be influenced by three parameters: voltage, impedance and phase angle changings. FACTS devices can influence these parameters [6]. It is important to choose the suitable kind of devices by mean to reach the required aims.

\[ \text{B}_{\text{SVC}} = B_\alpha \left(\frac{\alpha}{\pi}\right) + B_c \]  

(1)

\[ B_{\text{SVC, min}} < B_{\text{SVC}} < B_{\text{SVC, max}} \]  

(2)

SVC may have capacitive or inductive character to absorb or to provide reactive power \( Q_{\text{SVC}} \), respectively.

III. OPTIMAL LOCATION OF SHUNT FACTS DEVICES

The detection of sensor and weak nodes plays an important role for the optimal placement of shunt FACTS devices, which can be eliminated by analysis of network Jacobian matrix. This paper illustrates the significance of determining the minimum eigenvalue of the Jacobin matrix for evaluation of weak nodes und the interrelation between the minimum eigenvalue and the voltage.

The static voltage stability analysis is based on the modal analysis of the power flow Jacobian matrix, as shown in Equation (3). The dependence between the active \( \Delta P \) and reactive \( \Delta Q \) load variations at each nodes in the studied power system and the resulting nodal voltage phase \( \Delta \theta \) and magnitude \( \Delta U \) changing involves the Jacobian matrix inversion \( J^{-1} \), with a relationship given by [10]:

\[ \begin{bmatrix} \Delta \theta \\ \Delta U \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \]  

(3)

And:

\[ J = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial Q} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial U} \end{bmatrix} \]  

(4)

Due to realize an uncomplicated spectral analysis of Jacobian matrix for a large power system, it is meaningful to use the decomposed form of the Jacobian matrix [11] as following:

\[ J = WVW^T = \sum_{i=1}^{n} w_i \lambda_i v_i^T \]  

(5)

And accordingly:

\[ J^{-1} = V \Lambda^{-1} W^T = \sum_{i=1}^{n} \frac{v_i w_i^T}{\lambda_i} \]  

(6)

Where \( W = (w_1, w_2, ..., w_n) \) and \( V = (v_1, v_2, ..., v_n) \) are \( n \times n \) matrix with left and right eigen vector respectively. And \( \Lambda \) is a diagonal matrix of singular values \( \lambda_i \). It is essential: \( \lambda_1 < \lambda_2 < \lambda_3 \ldots \lambda_n \).

Using Equation (6) term (3) can be rewritten [11]:

Fig. 1. Power flow control and stability improvement [1]

Static Var compensator SVC is a controllable shunt element which is employed for voltage and reactive power control. This element can be used to improv stability in transient conditions and to increase transmission capability of networks [7], [8]. A possible structure of the SVC is given in Fig. 2.

Fig. 2. a) Structure of a SVC and b) model of a SVC

Static var compensator SVC is a shunt connected device which consists of a fixed capacitance in parallel with a thyristor controled reactor. This characteristic can be modelled by a shunt connected variable susceptance \( B_{\text{SVC}} \) with a lower bound \( B_{\text{SVC, min}} \) and an upper bound \( B_{\text{SVC, max}} \) which can be determined by the firing angle \( \alpha \) of the thyristors [2]:

...
\[
\begin{bmatrix}
\Delta \theta \\
\Delta P \\
\Delta U
\end{bmatrix}
- \frac{v_i w_i^T}{\lambda_1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

If the difference between the smallest singular value and the other ones is too small, then the term connected with this value has the largest influence on the voltage magnitude changes, namely:

\[
\begin{bmatrix}
\Delta \theta \\
\Delta P \\
\Delta U
\end{bmatrix}
= \frac{v_i w_i^T}{\lambda_1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

It can be seen that the voltage changing relative to reactive power changing is strongly dependent on the minimum singular value of the Jacobian matrix and this shows the significance of this value for the detection of sensor and weak nodes in large power systems. A node is called weak if its voltage magnitude is very sensitive against reactive power changes. According to Equation (6) a weak node has the sensitivity index for the detection of sensor and weak singular value of the Jacobian matrix and this shows the power changing is strongly dependent on the minimum eigenvalue of the Jacobian matrix conditioning. According to [11]:

\[
\frac{\partial \lambda_1}{\partial y_{\text{shunt},m}} = \pm v_{\text{rel}}^2
\]

Where \( y_{\text{shunt},m} \) and \( v_{\text{rel}} \) are admittance of the shunt at node \( m \) and right eigenvector of node \( m \) connected to minimum eigenvalue \( \lambda_1 \). The sensitivity of the eigenvalue \( \lambda_1 \) to the shunt admittance is called bus participation factor [12]. Accordingly to previous describing the sensitivity index for determination of optimal placement of shunt FACTS devices in the power systems can be defined as the component of bus participation factor corresponding to the minimum eigenvalue.

IV. DETERMINATION OF INFLUENCE AREA BY SENSITIVITY ANALYSIS

The influence of FACTS devices is limited on the area close to the devices. Injection the reactive power or changing the voltage at a certain bus bar modifies the voltages at other buses. But it is difficult to include the entire system into the process of Optimal Power Flow control by mean of determination of the set values. Therefore it is needed to perfome sensitivity analysis to determine the area on which the FACTS device has considerable influence and then this limited area will be involved in the Power Flow Optimization process. The area of influence is defined as an area for which the sensitivity is larger than a certain level.

The sensitivity of a variable \( y \) with respect to a control variable \( u \) is a measure for the impact of changes in \( u \) on the variable \( y \) [13], [14]. The sensitivity value is higher, if the control variable affects more on the considered system variable.

\[ U \cdot Q \] sensitivity analysis calculates the relation between voltage change and reactive power change:

\[
\Delta U = J_r^{-1} \cdot \Delta Q
\]

Where:

\[
\begin{align*}
\Delta U & : \text{incremental change in bus voltage magnitude} \\
\Delta Q & : \text{incremental change in bus reactive power injection} \\
J_r & : \text{reduced Jacobian matrix}
\end{align*}
\]

The elements of the inverse of the reduced Jacobian matrix are the \( U \cdot Q \) sensitivities. The diagonal components are the self sensitivities and the nondiagonal elements are the mutual sensitivities and are defined as SVC sensitivity factor \( K_{SV} \) for the determination of the influence area of SVC.

V. OPTIMAL POWER FLOW CONTROL

The aim of the OPF problem is to determine control setting of FACTS such that an objective function is minimized. Thereby the goal of optimization is to enhance the security level of the system. This means no overloading and no over- or undervoltages in the system. As economic aspect the minimization of active power losses is taken into account [14]. Thus, for a given system situation, for the optimal setting of FACTS the objective function can be defined as following:

\[
f(x) = \alpha \cdot \sum_i (U_i - U_{\text{ref},i})^2 + \beta \cdot \sum_j (I_j - I_{\text{lim},j})^2 + c \cdot \sum_j P_{\text{loss},j}
\]

Where \( U_i \) and \( U_{\text{ref}} \) are the voltage magnitude and the nominal voltage at bus \( i \), \( I_j \) is the working current in line \( j \), \( I_{\text{lim}} \) is the rated current of the line \( j \) and \( P_{\text{loss}} \) is the active power loss of the line \( j \). The weighting factors \( a, b \) and \( c \) will be chosen according to the importance of each term.

VI. TEST SYSTEM AND ANALYSIS OF THE OBJECTIVE FUNCTION

In this section a case study regarding optimal placement and setting of SVC will be introduced and analyzed. At first the sensitivity index for the detection of weak nodes will be calculated and the influence area of the optimal placed SVC will be determined using sensitivity analysis. Afterwards the objective function of the OPF problem will be analyzed. It will be evaluated for which part of the objectivities the shunt FACTS device is mainly responsible. And finally it will be shown how the voltage profile can be improved by the use of optimal located and sized shunt FACTS.

Fig. 3 presents the scheme of a test system. It consist of 57 nodes, 81 transmission lines and 7 generators connected at the nodes 1, 2, 3, 6, 8, 9 and 12. The generator connected at node 1 is assumed as a slake node. This test system is simulated with the software NEPLAN® and is used for all analysis in this paper.
Fig. 4 shows the base network condition without FACTS devices. This is a critical network situation with 21 undervoltages. The bus that is more sensitivity regarding to voltage magnitude or reactive power injection plays a decisive role for static stability analysis. By installing suitable reactive power compensation (SVC) at the weak bus a secured network voltage stability can be resulted. The calculated bus participation factors for the detection of sensor and weak nodes are shown in Fig. 5. It can be seen that the highest sensitivity is achieved for bus "31" which means that this node is a sensor node in the studied system and the optimal location of shunt FACTS device. The SVC installed at bus "31" improves the network situation in such a manner that no undervoltages appear in the system.

For the determination of influence area of the installed SVC at bus bar "31" the sensitivity analysis is performed and for each nodes of the studied system the sensitivities are calculated. Fig. 6 shows exemplary the sensitivity of 11 buses vs. the changing reactive power injection at the sensor bus bar "31".

It is obvious that for example the bus "2" and bus "57" have only low sensitivity relative to reactive power variation at bus "31" and therefor they are not concluded in the influence area. According the sensitivity analysis the influence area of the installed SVC at bus "31" can be determined. Fig. 7 demonstrated the influence area of the installed SVC at bus "31" in the test system. This includes all buses with a sensitivity higher than 0.06%/Mvar.
Furthermore the objective function of Optimal Power Flow control is analyzed. For a reduced problem only this influence area is considered for the optimization process. As mentioned in section V this function consists of three components. A certain type of FACTS is not able to influence all these components. In this paper it is exemplary evaluated for which part of the objectives the SVC’s are mainly responsible.

Therefore three different objectivities are defined and calculated \[15\]:

\[ t_1 = \frac{\sum_i (U_i - U_{i,ref})^2}{\sum_i (U_i - U_{i,ref})^2} \]  
\[ t_2 = \frac{\sum_j (I_j - I_{j,lim})^2}{\sum_j (I_j - I_{j,lim})^2} \]  
\[ t_3 = \frac{\sum_l P_{loss,lim}}{\sum_l P_{loss,lim}} \]  

Where \( k \) is the number of the included buses in the influence area and \( l \) is the number of the lines in the influence area. Fig. 8 shows the calculated objectivities \( t_1, t_2 \) and \( t_3 \) as a function of Q_{SVC} for the studied test system.

\[ f(x) = \sum_k (U_k - U_{k,ref})^2 \]  \[ f(x) = \sum_j P_{loss} \]

or:

For all three objective functions the minimum is reached for a Q_{SVC} equal 15 Mvar. This is the optimal set value of the SVC which is installed at bus "31" in the test system.

In the base case the voltage profile shows large deviations from the reference values. Undervoltage occurs at 21 buses (Fig. 5). As mentioned, the voltage profile can be influenced by SVC. Thus, the voltages are all in the acceptable range, between 95% and 105% with respect to the reference value.

**VII. CONCLUSION**

FACTS devices are a powerful technology that can solve many problems in power systems. They can influence power flows and voltages and therefore enhance the system security \[16\]. Voltage stability plays a major role in power systems. Static var compensator is a shunt connected FACTS device which is able to control the voltage profile in a system. It have been shown that for the determination of the optimal placement of shunt FACTS good results can be reached by analysis of the network Jacobian matrix. This fact is visible on the basis of a test system. The test system has a critical condition (21 undervoltages). Simulations are presented that the voltage profiles became more balanced with only one SVC in the test system.

For an optimal voltage control the set values of the FACTS controllers have to be selected correctly. Thus a method for the determination of the optimal set value based on optimal power flow has been introduced in this paper.

The objective function was analyzed in simulations. It was demonstrated that shunt FACTS devices are responsible for the parts with voltage deviations and active power losses.

The simultaneous use of several FACTS devices will be the subject for future studies. Additionally, the coordinated control
of several FACTS devices will be studied for the German 380 kV grid in order to investigate the performance in a more practical environment.

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REFERENCES


