Optimal Response of a Hydroelectric Power Plant with Bilateral Contracts

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Abstract—Deregulation and liberalization of electric power industry, among other things, has created new requirements for the market participants. The power system engineer, operator, and, in general, the market participant is being faced with requirements for which he does not have adequate training and the proper software tools. In this framework, among others, a pure hydro-generation company has to operate its hydro units, throughout the operating day, trying to fulfill the market clearing schedule or a bilateral contract. This paper addresses the optimal response of a hydroelectric power plant with bilateral contracts in order to maximize its profit from selling energy.

Index Terms—Hydroelectric power generation, hydroelectric generators, optimisation methods, electricity market.

I. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>total number of hours in scheduling period</td>
</tr>
<tr>
<td>$f_k$</td>
<td>cost function (penalty or benefit function according to the fulfillment of the contract in each hour $k$)</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>power deviation relative to the contracted one for each hour $k$</td>
</tr>
<tr>
<td>$U_j$</td>
<td>set of admissible decisions for plant units and reservoir; includes operation constraints, such as power limits and level limit constraints</td>
</tr>
<tr>
<td>$t_{jk}$</td>
<td>tariff type $j$ at hour $k$</td>
</tr>
<tr>
<td>$P^N_j$</td>
<td>plant nominal power</td>
</tr>
<tr>
<td>$P_{ia}$</td>
<td>power output of unit $i$ in hour $k$</td>
</tr>
<tr>
<td>$D_k$</td>
<td>contracted generation requirement in hour $k$</td>
</tr>
<tr>
<td>$J$</td>
<td>total number of units in hydro resources</td>
</tr>
<tr>
<td>$I$</td>
<td>total number of curves in power plant</td>
</tr>
<tr>
<td>$q_{ji}$</td>
<td>draft (through the powerhouse) in ($m^3 s^{-1}$) corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$q_{ji}^\text{max}$</td>
<td>maximum draft in ($m^3 s^{-1}$) corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$q_{ji}^\text{min}$</td>
<td>minimum draft in ($m^3 s^{-1}$) corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$P_{ji}$</td>
<td>power output in ($MW$) of unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$P^\text{max}_j$</td>
<td>maximum generating capacity in ($MW$) of unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$P^\text{min}_j$</td>
<td>minimum generating capacity in ($MW$) corresponding to unit $j$ in curve $i$</td>
</tr>
<tr>
<td>$P^\text{max}_j$</td>
<td>maximum generating capacity in ($MW$) corresponding to unit $j$ (whatever the curve $i$)</td>
</tr>
<tr>
<td>$P^\text{min}_j$</td>
<td>minimum generating capacity in ($MW$) of unit $j$ (whatever the curve $i$)</td>
</tr>
<tr>
<td>$P$</td>
<td>total power generated by plant (power demand) in ($MW$)</td>
</tr>
<tr>
<td>$Q$</td>
<td>total draft through all the committed units</td>
</tr>
<tr>
<td>$h_i$</td>
<td>head of curve $i$ in ($m$)</td>
</tr>
<tr>
<td>$u_j$</td>
<td>decision variable for unit $j$</td>
</tr>
<tr>
<td>$U_j$</td>
<td>set of admissible decisions for unit $j$</td>
</tr>
</tbody>
</table>

II. INTRODUCTION

Electricity industry restructuring has received government priorities worldwide while restructuring policies are debated at all levels internationally. The preliminary experiences have shown that the establishment of electricity market is going to be specific to legislations, cultures, economy, and electricity operations and practices in participating nations [1]-[3].

Portugal is also moving towards a competitive electricity market with the presumption that the competition will result in technological progresses, better services, higher efficiency and enhanced reliability, as well as less costly delivery of electricity to customers.

The electric utility deregulation and restructuring in Portugal has been implemented in a step-by-step way, and is now based on the existence of both Public Service Electric System (SEP) and Independent Electric System (SEI). The Non-binding Electric System (SENV) is part of SEI. The non-binding client is an individual or corporate body, the holder of an electric energy consumer installation, which has been authorized access to the SENV. The non-binding
Dynamic programming (DP) is among the earliest methods applied to the short-term hydro scheduling (STHS) problem [6]. Although DP can handle the nonconvex, nonlinear characteristics present in the hydro model, direct application of DP methods for cascaded hydro systems is very difficult to implement due to the well-known DP curse of dimensionality, more difficult to avoid in short-term than in long-term optimization without losing the accuracy needed in the model [7]. Artificial intelligence techniques have also been applied to the STHS problem [8], [9]. However, a significant computational effort is necessary to solve the problem for cascaded hydro systems. Also, due to the heuristics used in the search process only suboptimal solutions can be reached. A natural approach to STHS is to model the system as a network flow model, because of the underlying network structure subjacent in cascaded hydro systems [10], [11]. For cascaded hydro systems, as there are water linkage and electric connections among plants, the advantages of the network flow technique are salient. Hydroelectric power generation characteristics are often assumed as linear or piecewise linear in hydro scheduling models [11]. Accordingly, the solution procedures are based on linear programming (LP) or mixed-integer linear programming (MILP). LP is a well-known optimization method and standard software can be found commercially. MILP is very powerful for mathematical modeling and is applied successfully to solve large-size scheduling problem in power systems. Hence, MILP is becoming often used for STHS [12], where integer variables allow modeling of start-up costs and discrete hydro unit-commitment constraints. However, LP typically considers that hydroelectric power generation is linearly or piecewise-linearly dependent on water discharge, thus ignoring head-dependency to avoid nonlinearities. This is nowadays not appropriated for a realistic modeling of run-of-the-river hydro plants.

Hydro scheduling is in nature a nonlinear optimization problem. A nonlinear model expresses hydroelectric power generation characteristics more accurately and head-dependency on STHS can be taken into account. In the past, there were considerable computational difficulties to directly use nonlinear programming (NLP) methods to this sort of problem [13]. However, with the advancement in computing power and the development of more effective nonlinear solvers in recent years, this disadvantage has much less influence. We have shown as a recently new contribution, [14], [15], that this disadvantage is mitigated by applying a nonlinear approach to a realistically-sized hydro system, respectively, with three and seven cascaded reservoirs, which was not possible with earlier approaches and computational resources.

Although, this recent developments, mainly based on the short-term hydro scheduling problem, can be very important to elaborate a daily operation plan of its hydro resources in order to assess the available energy that could be delivered to the grid. The paper presents the main problem and its mathematical formulation, as well as the computational adopted method for solving it. After, some illustration results are presented and finally some conclusions are taken.
III. PROBLEM FORMULATION

A non-binding producer, which has established a bilateral contract, must put power into the grid (assuming that there is technical feasibility) that the non-binding clients will consume in order to fulfill the contract. That contract constitutes, for the non-binding producer, the exploitation program that, by rule, consists of the power that he must deliver to the network, each hour, during a day. If he doesn’t fulfill the contract (by default or by excess) it will incur in costs or incomes associated with deviations. These deviations result from the difference between the contracted values and those recorded in practice, and are calculated based on nominal power $P_s$, which the producer has installed. In this case, a non-binding producer is responsible for a hydroelectric power plant whose exploitation he intends to manage optimally along the day (duration of the contract). So, the optimal exploitation problem of the plant includes the contracted load profile, the penalties for production deviations from the contracted profile and the constraints associated with the hydroelectric power plant. The resolution of this problem allows achieving the optimal production profile. In this case, the goal isn’t to meet the load profile, but to minimize costs and, if possible, to achieve production benefits. Thus, the formulation of the problem $\mathcal{P}$ is the following:

$$\mathcal{P} \quad \text{Min} \sum_{k=1}^{K} f_k(\delta_k)$$  \hspace{1cm} (1)

subject to

$$\delta_k \in \mathcal{U}_k$$  \hspace{1cm} (2)

The objective function of problem $\mathcal{P}$ results from a sum of functions, a function for each hour $k$, and each function $f_k : \mathbb{R} \to \mathbb{R}$ is defined as follows:

$$f_k(\delta_k) = \begin{cases} -\frac{\beta}{\alpha} t_{ik} & \text{if } \delta_k < -\beta P_s \\ -\frac{\alpha}{\beta} t_{ik} & \text{if } -\beta P_s \leq \delta_k < -\alpha P_s \\ -\delta_k t_{ik} & \text{if } -\alpha P_s \leq \delta_k \leq \alpha P_s \\ -\delta_k t_{ik} & \text{if } \alpha P_s < \delta_k < \beta P_s \\ 0 & \text{if } \beta P_s < \delta_k \end{cases}$$  \hspace{1cm} (3)

where

$$\beta > \alpha \quad \text{for } \beta \in \mathbb{R}^+ \quad \text{and } \alpha \in \mathbb{R}^+$$  \hspace{1cm} (4)

and

$$\delta_k = \sum_{j=1}^{J} P_{ij} - D_k$$  \hspace{1cm} (5)

As mentioned, the objective function is a penalty (or benefit) function according to the fulfillment of the contract in each hour $k$. The penalty or benefit depends on the deviation and it’s always linear. The angular coefficients of each penalty are given by $t_{ik}$, and obey the following relation: $t_{1k} > t_{2k} > t_{3k}$ . Thus, as illustrated in Fig. 1, for deviations smaller than $\alpha \times 100\%$ we get benefit or penalty (according to the tariff $t_{1k}$); for negative deviations among $\alpha \times 100\%$ and $\beta \times 100\%$ the penalty is higher (according to the tariff $t_{2k}$) and it’s still aggravated for negative deviations above $\beta \times 100\%$ (according to the tariff $t_{3k}$); for positive deviations above $\alpha \times 100\%$ there is no benefit or penalty.

As shown, each of the partial functions $f_k$ is discontinuous, nonlinear and nonconvex. These properties raise difficulties to achieve the solution of the problem $\mathcal{P}$ and require an optimization beyond the field of conventional nonlinear programming. The method used here to overcome this difficulty is an implicit enumeration method, in which all possible decisions are tested and the best decisions are then chosen. This ensures that the results are optimal and globally optimal. The disadvantage of this method comes from the requirement to work in a discretized space, demanding for more memory and runtime. To avoid that the runtime make this method impracticable, it requires a robust implementation of the algorithm that is based, essentially, in an efficient data structure, computed offline. Thus, it is possible to reduce operations that, for being repetitive, lead to a significantly increased runtime.

When solving the problem $\mathcal{P}$, it is essential to know, at each level and for each possible draft to turbine, the best combination of unities that corresponds to the maximum energetic efficiency (for a given head and draft) and the power allocated to each unit. This problem is a unit commitment problem in the hydro plants and will be presented in the next section.

Fig. 1. Illustration of the objective function. $f_k$—penalty function according to the fulfillment of the contract in each hour $k$. 

IV. POWER HOUSE I/O CURVES CONSIDERING HEAD DEPENDENCY

The hydro generation model is either unit or plant-based. For a more accurate approach, each individual unit in a plant is treated separately, which yields a hydro unit commitment problem. The hydro unit commitment leads to a solution that makes possible the adoption of an aggregated plant concept. Thus, the units in a hydro plant are aggregated as one equivalent plant, while maintaining all information about the status of each individual unit – the unit commitment in the power plant can change, according to the head and the water flow to achieve optimal solution. The electric power generated is computed as a function of water flow, depending on hydro unit input/output (I/O) characteristic associated with the corresponding head. The dispatch of head dependent hydro units (set of characteristic curves, each one for a constant value of electric generated power, for each hydro power plant) incorporates water flow unit limits, unit generated power limits and the head dependency effect. In particularly, this problem assumes a great complexity when the units in a power plant are different from each other, mainly because some of them saw its capacity increased, and because the objective function is non-linear and non convex. For these reasons the problem solution imposes an optimization out of conventional non-linear programming (increasing the runtime). The advantage of using the aggregated plant concept is that it can be done offline, reducing significantly the time required in the optimization process in hourly hydro resource scheduling, for energy.

A. Mathematical Formulation

Given the imposed constraints, those required for each unit and those connected with all units, a proper unit commitment decision must be chosen and must be optimal from the economic benefit point of view. This problem involves, by one way, the statement of all possible decisions and the value associated with each of them, and by another way, the strategy analysis used to achieve the optimal solution. Thus, the problem formulation brings another problem, of mathematical programming, non-linear, described as follows.

Consider a hydro power plant with \( J \) units. Each unit is characterized by three variables: power, water flow and head. If one of these variables is kept constant - let be the head - each unit \( j \) is characterized by a set of curves. The number of curves \( I \) is as big as bigger are the discretization levels, assumed for the head.

Each curve \( i \) of unit \( j \), can be represented as a function of the generated power and the net head:

\[
q_{ji} = f(p_{ji}, h_i)
\]  

(6)

with \( i = 1, \ldots, I \) and \( j = 1, \ldots, J \)

The goodness of different possible decisions is made based on an established scale that characterizes each solution. This measurement scale is obtained from a function – objective function. The objective function that better fits the problem under analysis is the water flow through the turbines within the powerhouse (the water flow represents the operating cost).

Thus, expression (6) is a cost operation function, and the main problem to determinate the dispatch of head dependent hydro units (power plant characteristic curves) is related to the optimal unit commitment problem, and can be presented as follows.

For a set of units within a hydro power plant, minimize the operating cost, according to:

- power demand – constraint connected with all units
- minimum and maximum generating capacity of each unit depending on head – constraint on individual curve
- minimum and maximum generating capacity of each unit independently on head – constraint on individual unit

So, the hydro unit commitment problem, for each curve \( i \), can be written as:

\[
\text{Min}_u \quad \left( \sum_{j=1}^J q_{ji}(p_{ji}, h_i, u_j) \right)
\]  

(7)

subject to:

\[
\sum_{j=1}^n p_{ji} = P
\]  

(8)

\[
p_{j}^{\min}(h_i) < p_{ji} < p_{j}^{\max}(h_i) \cap p_{j}^{\min} < p_{j} < p_{j}^{\max}
\]  

(9)

where:

\[
 u_j \in U_j \quad j = 1, \ldots, J
\]  

(10)

Expression (7) represents the total value of water flow and indicates that for a specific value of generated power \( P \), with head \( h_i \), the water flow depends on the unit’s dispatch for the considered unit commitment. Expression (8) represents the generated power by the plant, for the considered unit commitment. Expression (9) is the result of considering the minimum and maximum generating capacity of unit \( j \) in curve \( i \), together with the minimum and maximum generating capacity of unit \( j \) whatever the curve is. The expression (10) represents the resource feasibility set.

B. Illustration Results—Without Considering the Elevation of the Downstream Head

As an illustration, we consider the case study of a small hydro power plant with six units, G1-G4 (identical units), G5 and G6. Each unit is characterized by eight curves, \( I = 8 \), and the relation between heads is given by \( h_{i+1} > h_i \) with \( i = 1, \ldots, 8 \). In this example, the problem solution allows to obtain eight characteristic curves for the power plant – the same number of curves that characterizes each unit, without considering the elevation of the downstream head with the water flow through powerhouse.
Fig. 2 and Fig. 3 show the characteristic curves of the hydro power plant, for constant values of head and for constant values of power, respectively.

For any value of power $P$ generated by the hydro power plant, and for the considered values of head $h$, the water flow $Q$ is minimum, defining the unit commitment ($Q$ is the total water flow through the committed units). In the case of unit commitment involving a combination of units, the level of power generation is different for each one of them. Fig. 2 shows the total values of power and water flows.

Note that a discontinuity exists, near low power area, caused by the transitions between different unit commitments. This fact results from both the different characteristics of each unit and the generating capacity limits. Except for the critical area, the curves have a smooth and continuous evolution.

Fig. 3 shows the characteristic curves of the hydro power plant for a constant value of generated power. This figure shows the increase in water flow needed to generate the same value of power with the decrease in head. It can be seen that for some values of power, the unit commitment changes, according to the head and the water flow to achieve optimal solution. The critical area referred above can also be seen near low power values. Also, Fig. 3 shows the obtained different unit commitments with different colors. Each color represents a different combination of units. Note that is possible to obtain up to nine different commitments for the same generated power, up to five different commitments for the same water flow and up to ten different commitments for the same head.

V. ILLUSTRATION RESULTS

The numerical results of the problem's solution $(\mathcal{P})$ is now presented for the case considered in previous section. In the resolution of this problem the goal is not to satisfy precisely the load profile, but to minimize costs and, if possible, to obtain production benefits. That is, with the available data (inflow to the reservoir every hour, initial and final reservoir levels) and considering all the problem constraints, the question that we intend to see answered is the following: what is the exploitation profile that allows to achieve this goal?

The resolution of the problem answers optimally to this question, as we will illustrate below.

As mentioned in Section III, the objective function of problem $(\mathcal{P})$ results from a sum of functions, a function for each hour $k$, and it is a function described in terms of more than one expression, depending on the value of the parameters $\alpha$ and $\beta$. In this illustration example the following values for parameters $\alpha = 0.05$ and $\beta = 0.15$ was considered.

Fig. 4 shows that we can, according to the deviation, incur into costs or profits. We get benefits when producing in excess (not exceeding the level of 5%) and incur into costs, as the deviation, when producing in default. As the energy at peak and full hours is more valued, it’s also at these times that we seek to obtain profit. Note that we always get benefit at peak and full hours, with the exception of three hours. It should also be noted that the deviation follows the threshold levels in the bilateral contract ($\alpha$ and $\beta$ parameters) – these levels correspond to the points of discontinuity of the objective function and represent a leap in terms of penalty or benefit.
Regarding this example, as illustrated in Fig. 4, it was preferable to operate, in percentage of deviation δ, on the threshold of the level -15% at off-peak hours, forcing the run, for three hours, in the threshold of the level -5% (in off-peak hours the deviation didn’t get below -15% and in the full hours the deviation didn’t get below -5%). Producing in these threshold levels result in existence of discontinuities in the objective function (penalty leaps). These points of discontinuity (critical points) play a key role in the economic exploitation of this plant, justifying the optimization method used here. The use of optimization algorithms based on nonlinear programming would require that the objective function was convex and, thus, the results would be either very limited or out of the points of discontinuity.

![Fig. 4 Minimum cost and deviation function. The bars with the three shades of gray represent: (1) in the x-axis the hourly tariff; light gray – off-peak hours tariff, intermediate grey - full hours tariff and dark gray - peak hours tariff and (2) in the y-axis tariff according to the deviation and for each hour \( k \); light gray – deviation tariff \( t_{\delta,1} \), in percentage, in the interval \([-5, 5]\), intermediate grey – deviation tariff \( t_{\delta,2} \), in percentage, in the interval \([-15, 5]\) and dark gray – deviation tariff \( t_{\delta,4} \), in percentage, in the interval \([-\infty, -15]\).](image)

VI. CONCLUSION

This paper addressed the optimal response of a hydroelectric power plant with bilateral contracts in order to maximize its profit from selling energy. In this new scenario the solution of this problem was obtained. In particular, its formulation and its solution were illustrated.

The problem of unit allocation in hydro power plants was formulated and solved. The solution of this problem has enabled to achieve the optimal units allocation. Thus, a set of characteristic curves were obtained for the plant, which correspond to maximum energetic efficiency, and allows knowing all the values of power output that the plant can produce, for all values of head, which units that must be used and the respective draft.

The obtained results allowed to: (1) get the profile of the optimal exploitation for a specific bilateral contract, between a non-binding producer and a non-binding client; (2) show how the optimal exploitation is done in the context of the restructuring, the new requirements and the new behaviors and (3) show that the exploitation of a resource, in this new framework, obeys to different criteria from the usually used, which results in changes in how to operate the plant, always difficult to achieve and implement, because it is different from the traditional way.

REFERENCES