A Model Based Approach for Pipeline Monitoring and Leak Locating

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Abstract—A fluid flowing through a pipeline is a complex phenomenon, depending on many physical factors. Developing efficient methods for system monitoring and leak locating start with a good understanding of the pipeline. The modeling technique becomes a very useful tool, allowing a separate analysis of the different hypostases describing the process. This paper investigates the problem of using a reduced lumped parameters model for the pipeline system. Both static and dynamic regimes are discussed. The final goal is oriented on testing the acoustic methods on the resulted model, given the fact that the pipeline is a multipath, noise-correlated environment. An equivalent laboratory experimental model is used for comparison.

I. INTRODUCTION

Pipeline systems have represented since a long time a practical way for transporting fluid from one point to another. One of the most common problems encountered here is the risk of the accidental leaks, which in turn can cause significant economic damage, affect the environment, or sometimes can be a threat to the public safety. Until now, no general, efficient method for detecting and locating leaks was found.

Most of the techniques are based on monitoring the state variables (pressure, flow rate, temperature, etc.), measured at different points on the pipeline. The most known method uses the volume balance law at the system’s inlet and outlet, [1], [2] and can be used only for detection. In this case, most of the measurements have to be taken off-line, by isolating the pipeline under observation. Likewise, this method is adequate for detecting large leaks, and generally requires a large time interval for this purpose.

Some other advanced methods use the mathematical model of the pipeline system, based on the mass, momentum and energy conservation laws [3], [4], [5]. In this respect, numerical methods are involved for solving the resulting partial differential equations. Among the most known of them, are situated the method of characteristics and the prediction-correction method [3], [4]. According to these techniques, the pipeline is divided into smaller segments. The computation is performed gradually, based on the values from the previous time moment and the boundary conditions. The results are compared with the measured values, transmitted periodically from a data acquisition system. The difference between the measured and the computed values provide information regarding the system behaviour. However, these methods need to take into account the uncertainties regarding the friction coefficient, the fluid density variation, etc. Therefore, the amount of computation time required by these techniques is usually large due to the space-time discretization and the complexity of the system. Also, they use a large number of observation instruments for the process variables, some of them intrusive, which sometimes are not available.

Recently, the acoustical methods have become widely used for leak detection and location [1], [6], [7], [8], [9]. They are based on monitoring the acoustic noise produced by the fluid transiting the pipeline. Here, the observation equipment employs non-intrusive sensitive mechanisms like piezoelectric elements which perceive the flow noise and vibration. The most known acoustic technique is based on estimating the time delay at which the acoustic signal reaches at the two separate locations on the pipeline [1], [6], [7]. With this information, a leak can be located providing the wave propagation velocity along the pipe and the distance between the sensors.

Therefore, the leak locating problem reduces to the problem of time delay estimation. There is a rich literature which deals with this subject with numerous applications in military, industry, medicine, etc. [10]. A typical method for estimating the time delay consists of computing the cross-correlation function [10]. The time delay corresponds to the argument of the cross-correlation function’s maximum. In practice, this parameter is affected by errors, providing that the pipeline system operates frequently in environments characterized by small signal-to-noise ratios (SNR). There are already known in the literature many reference techniques which have been developed to extract the information regarding the time delay from the background noise [10]. All these algorithms are generally based on the assumption that the acquired signals are described by ideal aspects (i.e. the signals are white, Gaussian, stationary random processes, and the additive noises are uncorrelated with the primary source and with each other).

For the pipeline system, these assumptions are not always carried out. The experience shows that the signals can be non-stationary both in their mean and variance, due to the pipeline modal components, a turbulent flow, or any non-stationary conditions present at the acquisition time. On the other hand, especially for small-scale installations, the background noise has a correlated component, caused by the pipeline main stream, which sometimes can’t be neglected. Also, in this case, the effect of the multipath propagation is considerable.
due to the inherent echoes that are present in the system. This paper aims to continue the investigation in this field, using a modeling approach of the flowing process. In this respect, the pipeline is approximated with a segmented lumped parameters system, analog to a transmission line. Both static and dynamic regimes are discussed. It is noted the similarity of the pipeline is approximated with a segmented lumped parameters modeling approach of the flowing process. In this respect, the paper aims to continue the investigation in this field, using a due to the inherent echoes that are present in the system. This technique becomes an important tool, providing a better understanding of the pipeline system in order to choose the most effective methods for the given configuration. An equivalent experimental model is used for results comparison.

II. THE PIPELINE MATHEMATICAL MODEL

A. Governing Equations

The relationships that describe a fluid flowing through a pipeline, known in the literature as the water-hammer equations [3], [11], are obtained by applying the mass and momentum conservation laws. Assuming a horizontal straight pipeline segment, with constant circular cross-sectional area \( A \) and diameter \( d \) transited by a compressible fluid with density \( \rho \), at constant temperature, the system’s governing equations can be written as (SI units):

\[
\begin{align*}
\frac{\partial q}{\partial x} &= -A \frac{\partial p}{\partial t} \\
\frac{\partial q}{\partial t} + A \frac{\partial p}{\partial x} + q \frac{\partial q}{\partial x} &= 0
\end{align*}
\]

(1)

where \( p=p(x,t) \) and \( q=q(x,t) \) are the state variables, pressure and flow rate respectively, both depending on location, \( x \) and time, \( t \). Also, \( c \) is the wave velocity, and \( f \) is the friction factor, assumed to be constant along the pipe. Depending on the Reynolds number [11] value, the flow can be categorized as laminar or turbulent and the friction coefficient is computed according to certain empirical formulas or by using the Moody diagram [12].

For solving the water-hammer equations, numerical methods are employed, in connection with the boundary and the initial conditions of the given system. The most known technique, called the method of characteristics [4], transforms the partial differential equations (1) into a pair of total differential equations as in:

\[
\begin{align*}
\pm \frac{1}{\rho c} \frac{\partial p}{\partial t} + \frac{L}{A} \frac{\partial q}{\partial t} + f \frac{\partial q}{\partial x} &= 0 \\
\frac{\partial x}{\partial t} &= \pm c.
\end{align*}
\]

(2)

The pipeline is discretized in both space and time dimensions. Integrating (2) along the positive and negative diagonals (characteristics) [4], the current state variables pressure and flow rate at the \( i \)-th node can be computed from the adjacent points values. The numerical methods usually employ a lot of monitoring instruments along the pipeline and a large amount of computation time in order to find the values for pressure and flow at each node, at every time moment. Sometimes it is important to use simplified models, for gaining more flexibility in analyzing, understanding and dealing with the complex circumstances taking place in the system. The task of finding optimal solutions for pipeline monitoring and leak detection is made easier.

B. The Lumped Parameter Model

An alternative simplified approach consists in modeling the pipeline as a segmented system, where each segment is approximated with a lumped parameter sub-system [13]. When the flow rate variation \( dq \) is much smaller than the nominal flow rate \( Q_0 \), the following relationship can be assumed:

\[
q = Q_0 + dq \equiv Q_0
\]

(3)

In this situation, (1) become similar to the telegrapher’s equations specific for the transmission lines. For this case, each pipeline segment of length \( \Delta x \) can be modeled as an \( RLC \) type quadrupole where the three parameters are similar to the electric resistance \( R \), inductance \( L \) and capacitance \( C \) per unit length, respectively. Here, \( R \) and \( L \) are series elements while \( C \) is a parallel element. The relationships that describe them are straightforward, providing that the pressure and flow rate variables are replaced in the new representation by voltage and current:

\[
\begin{align*}
R &= \frac{f \rho}{2dA^2} Q_0; \quad L = \frac{\rho}{A}; \quad C = \frac{A}{\rho c^2}
\end{align*}
\]

(4)

The number of segments needed to assure a good compromise between the model’s precision and the computation amount, is chosen following the rule of thumb that \( \Delta x < \frac{\lambda}{10} \), where \( \lambda \) denotes the minimum wave length of the perturbation signal [14]. Therefore, the number of segments, \( nseg \) will satisfy the following relationship:

\[
nseg > 10 \cdot l / \lambda_{\text{min}}
\]

(5)

where \( l \) is the total pipeline length.

It follows that the theory developed for the transmission lines can be applied also for this approximated pipeline model. The load impedance \( Z_L \) will depend on the downstream equipment, which usually is represented by a valve. If \( Y \) is the percent of the output valve that is open (100% representing maximum opening), then:

\[
Z_L = K \rho Q_0 \sqrt{2A^2 Y^2}
\]

(6)

where \( K \) is the minor loss coefficient for the valve [11], [15] and \( Q_0 \) is the nominal output flow rate (at \( x=l \)). Using the transmission line analogy, neglecting the loss due to friction, the characteristic impedance \( Z_0 \) can be approximated as:

\[
Z_0 = \sqrt{L/C} = \rho c / A
\]

(7)
The normalized load impedance can be put in the form:

\[
Z_l \frac{d}{dz} Z_I = K \frac{\rho Q_1}{2 A^2 Y^2} \frac{\partial A}{\partial \gamma} - \frac{1}{\rho c} \frac{\partial Q_1}{\partial z} = K \frac{\partial A}{\partial \gamma} - \frac{1}{2 A Y^2 c} = \frac{\partial \gamma}{\partial z}
\]

where the factor \( m^2 = v/c \) (known as the Mach number) showing the ratio between the fluid and the wave velocity, is usually very small in this type of applications. Therefore, it can be observed that when the downstream valve has maximum opening, the load impedance is much smaller than the characteristic impedance of the pipe. This situation can be compared with a transmission line with a shorted load. On the other hand, when the downstream valve is completely closed, the pipeline system is similar to an open load transmission line. Also, in the same manner, a leak in an arbitrary distance \( x \) from the input, can be modeled as a parallel impedance \( Z_p \) on the line (fig.1).

III. THE EXPERIMENTAL AND THE SIMULATION MODEL

For testing the performance of different monitoring techniques, an experimental laboratory model of relatively small size (comparable with the size of a 30 m² room) was implemented [16]. The fluid used in this experiment was cold water, at about 20°C. The chosen configuration was bended, with a total length of 12.82 meters. The pipeline was made of steel, with a circular section, and 2.54 centimeters diameter. The leaks were simulated by two faucets (fa and Fb) placed in the middle and towards the pipeline end. An additional faucet was placed at the system’s output, enabling the two extreme boundary conditions: open or closed downstream end. The acoustic pipeline signals were acquired simultaneously by two piezoelectric transducers (accelerometers) KD Radebeul. The resulted signals were passed through a pair of amplifiers M60T with adjustable gain between 40 and 60 dB and anti-aliasing low pass filters. The digital conversion of the received signals from the experimental installation was performed by a dSPACE DS1102 board connected to a PC. The sampling frequency was set to 25 kHz, sufficient for covering the acoustic domain of the received signals. The measuring points were placed at equally distributed intervals of 0.3 m along the pipe. The acquired signals were further processed through Matlab®/Simulink® environment.

Based on the experimental set up described above, an equivalent simulation lumped parameters model was implemented using Matlab®/Simulink® and the Simscape™ toolbox. The number of the segments was varied, depending on the maximum frequency employed in the study, according to (5). The state variables, pressure and flow rate were monitored in the same measuring points as in the experimental set up. The leaks and the pipeline end were modeled with hydraulic orifice blocks, with programmed varying areas. The hydraulic reference was considered the atmospheric pressure. Both static and dynamic studies were performed and the obtained model proved to be a valuable tool for understanding the system.

IV. THE STATIC REGIME

Assuming a constant pressure \( P_0 \) at the pipeline input \( x=0 \), in the absence of a leak, the state variables can be computed in terms of the upstream and downstream end pressures, \( P_0 \) and \( P_1 \):

\[
q(x) = Q_0 = \sqrt{2 \rho \frac{d}{dx} \left( P_1 - P_0 \right)}, \quad p(x) = P_0 + \left( P_1 - P_0 \right) \cdot x / l.
\]

In the presence of a leak at \( x=x_F \), with a discharge \( Q_F \), the flow rate will perform a stepwise discontinuity (fig.2).

Integrating over the two sub-domains, knowing the boundary pressures at both pipe ends \( P_0 \) and \( P_1 \), setting the condition of continuity at \( x=x_F \), the static pressure expression in the presence of a leak becomes:

\[
p(x) = \begin{cases} 
\frac{\rho Q_0^2}{2 l^2} \cdot x + C_1 & \text{for } x \in [0,x_F] \\
\frac{\rho Q_1^2}{2 l^2} \cdot (Q_0 - Q_F)^2 \cdot x + C_2 & \text{for } x \in [x_F,l]
\end{cases}
\]

where the free terms \( C_1 \) and \( C_2 \) are:

\[
C_1 = P_0, \quad C_2 = P_0 - \frac{\rho Q_1^2}{2 l^2} x_F^2 + \frac{\rho Q_1}{2 l^2} \cdot (Q_1 - Q_F)^2 x_F^2
\]

In the presence of a leak, the pressure graph is composed of two lines with different slopes, proportional with \( Q_F^2 \), upstream, and with \( (Q_1 - Q_F)^2 \) downstream, respectively. Pressure and flow profiles can be deduced in the static regime for other cases of interest (e.g. a constant flow rate source at the input, or a closed pipeline end), following a similar procedure. It can be remarked that these methods are more suitable for detecting large leaks, which produce significant variations in the state variables profile, rather than the small ones. In addition, if the installation contains other elements like bends, junctions, etc., the monitoring operation is hardened due to the additional deformations in the variable profiles, as illustrated in fig.3.
pipeline sections that are joined together. An interesting
impedance (i.e. shown [18], [19] that in general, for a non-matched output
theory from the transmission lines can be applied. It can be
perturbations relative to the operating point as in (3), the
closed, shorted output. Conversely, when the downstream end is
Γ
reflection coefficient at the pipeline end. Therefore, when the
impedances [19].
The characteristic impedance (7), the resulted reflection
discontinuity [17]. For a pipeline junction, the reflection
coefficient can be written as:

\[ \Gamma = \frac{A_I/c_I - A_O/c_O}{A_O/c_O + A_I/c_I} \]  

(12)

where the subscript indices 0 and I indicate the two different
pipeline sections that are joined together. An interesting
remark is obtained when dividing (12) by \( \rho \) and substituting
the characteristic impedance (7), the resulted reflection
coefficient has a similar form to the one encountered at the
junction of two transmission lines with different characteristic
impedances [19].

The above relationship can be extrapolated to compute the
reflection coefficient at the pipeline end. Therefore, when the
downstream valve is open, \( A_I \) tends to infinite in (12) and thus
\( \Gamma_{\text{infinite}} = 1 \), situation similar to a transmission line with a
shorted output. Conversely, when the downstream end is
closed, \( A_I = 0 \), and evidently \( \Gamma = 1 \), similar to a transmission line with an open load. Therefore, for small amplitude
perturbations relative to the operating point as in (3), the
theory from the transmission lines can be applied. It can be
shown [18], [19] that in general, for a non-matched output
impedance (i.e. \( Z_I \) is different from \( Z_0 \)), the pressure consists of
two waves that travel in the same time in opposite
directions on the pipeline. This interference will lead to
standing waves, exhibiting nodes and antinodes correlated
with the pipeline ends. Defining the ratio between the
maximum and minimum pressure amplitudes, the standing
wave ratio factor (SWR) at the distance \( x \) from the pipeline
input, can be derived by analogy with the transmission lines:

\[ \text{SWR} = \begin{cases} \left( \frac{A_O}{A_I} \right) \left( \frac{c_I}{c_O} \right) & \text{for } A_O/A_I \geq c_0/c_I \\ \left( \frac{A_I}{A_O} \right) \left( \frac{c_O}{c_I} \right) & \text{for } A_O/A_I < c_0/c_I \end{cases} \]  

(13)

Assuming a lossless pipeline, and a matched input impedance
\( Z_0 = Z_{in} \), (fig.1 with \( Z_0 \) infinite) an equation in \( P_{in} \) is obtained,
which finally leads to the following expression for the output
wave speed \( c_I \), in terms of the pipeline end area ratio \( A_0/A_I \):

\[ c_I = \frac{c_0}{2} \frac{A_I}{A_0} + \left( \frac{c_0}{2} \frac{A_I}{A_0} \right)^2 \frac{K}{2 \rho} P_0. \]  

(14)

The theoretical variation with \( A_0/A_I \) of SWR and \( \Gamma \) obtained
by substituting (14) in (13) and (12) respectively, is illustrated
in fig.4, together with the results obtained by simulation. It
can be observed practically a matched output impedance for
an area ratio around 34, for the considered model.

In the presence of a leak, the reflection coefficient can be
derived from (2) and the mass and energy conservation laws:

\[ \Gamma \equiv \frac{A_I/c_I}{2\left(\frac{A_O}{c_0}\right) + \left(\frac{A_I}{c_2}\right)} \]  

(15)

Here \( A_I \) and \( c_2 \) denote the leak area and the wave velocity at the
leak output, respectively. The SWR factor becomes, in this case:

\[ \text{SWR} \equiv 1 + \left( \frac{A_I}{A_O} \right) \left( \frac{c_0}{c_2} \right). \]  

(16)

When the leak area \( A_I \) is very small then \( \Gamma \to 0 \) and

\[ \text{SWR} \to \infty \]  

tending to a matched impedance, as expected. On the other hand, when the leak area is increased such
that \( 2 \left( \frac{A_O}{c_0} \right) \ll \left( \frac{A_I}{c_2} \right) \), then \( \Gamma \to -1 \) and

\[ \text{SWR} \to \infty \]

Following a similar reasoning with the previous case, the
input pressure \( P_{in} \) and the wave speed at the leak output \( c_2 \) can be
derived, then based on (15) and (16), \( \Gamma \) and SWR factors.

Therefore, when a leak occurs, the system can be viewed
from the input as a shortened pipeline of length \( x \), (equal to the
distance to the leak), terminated on the parallel
impedances \( Z_I \) and the equivalent input impedance at the leak,
\( Z_0(x) \) (fig.1). Another useful remark concerns the pressure
spectrum at the pipeline input. In the absence of a leak, the

![Figure 3: Pressure profile of a pipeline with two bends and a leak](image)

![Figure 4: Standing wave ratio and reflection coefficient dependence of downstream area ratio](image)

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first spectral component corresponds to a pipeline of \( \lambda/4 \) length. The standing wave configuration depends on the downstream end condition. A leak occurrence at an arbitrary distance \( x \), produces a discontinuity in the pipeline system. Assuming a matched input impedance, this defect will change the standing wave distribution in the system, resulting a pressure node at the leak, which will divide the pipeline into two regions of type \( \lambda/4 \) upstream, and \( \lambda/4 \) or \( \lambda/2 \), depending on the output condition, downstream, respectively. This observation can be used as a leak detection method by extracting the information regarding the distance \( x \) to the leak, from the input pressure frequency components, \( F_{pi} \):

\[
F_{pi} = (2k + l)c/(4x) \quad i = 1, 2, \ldots
\]  

(17)

Using an input sinusoidal command signal \( p_0 \) (fig.1), of small amplitude and with a linear increasing frequency (a “chirp” signal), the natural frequency domain of the pipeline will be swept. Multiple stationary waves will be produced at the time moments at which the command signal frequency coincides with each of the pipeline own frequencies. Fig.5 presents the pressure stationary waves distribution for the discussed model in the presence of a leak, obtained by simulation. If the leak position is increased gradually on the pipeline, an inverse dependence with the distance of the pressure harmonics can be measured at the pipeline input, in accordance with (17). Fig.6 presents the simulation results for the first five resonant frequencies (\( f_{p1} \ldots f_{p5} \)) of the studied pipeline, in the presence of loss, as a function of distance (in the upper part) and their variation with the leak area, at a fixed distance from the input (in the bottom part). The results indicate also a small decrease of these frequencies with the leak opening.

VI. PIPE FLOW NOISE AND VIBRATION MODELING

Using the pipeline model previously discussed, a convenient way can be found for implementing the acoustical methods for leak detection, based on time delay estimation.

![Fig.5 Standing wave distribution in the presence of a leak at x=780 cm when an input chirp signal is applied](image)

This can be done by viewing the noise and vibration due to the fluid flow as a dynamic regime, produced by random noise sources. These sources, associated with the pipeline main stream and the leak, can be modeled as orifices (impedances) with random varying values of very small amplitude around their nominal values. The random sources can be represented by band limited white noises or even samples of real signals, acquired in the experimental installation. In this way, the effects due to the multipath propagation in the pipeline system can be evaluated. Several different studies can also be completed regarding the algorithms behavior in a correlated or a non-stationary noise environment, conditions that are generally in contradiction with the assumptions of the classical techniques. Fig.7 shows two typical cross-power spectral densities corresponding respectively to a simulated and a real signal, measured in the same locations on the pipeline model. Also, fig.8 presents comparatively a typical cross-correlation function (CCF) of the simulated and experimental signals, estimated using the phase transform (PHAT) processor [10]. The pre-whitening procedure [20] is also employed for an increased estimation accuracy (fig.8, bottom). Finally, fig.9 presents comparatively the time delays estimated along the pipeline, using the simulated and experimental signals. The outlying points can be explained by the specific conditions attained in the pipeline system, that divert from the ideal assumptions (multipath propagation, correlated noise, non-stationary conditions present at the recording time like traffic, turbulent flow, etc.)

VII. CONCLUSIONS

Pipeline models are important tools for understanding the system complexity, in order to find the optimum solutions for leak detection and location. The mathematical model that describes a fluid flowing through a pipeline is represented by a non-linear set of partial differential equations, which requires a large amount of computation time for solving. In practice, the possibility of obtaining fast results through simulation is desired, as a foundation for real time monitoring.
This paper uses a simplifying approach for modeling the pipeline system, based on the analogy with the transmission lines, with the emphasis on wave propagation. A good similarity of the reflection coefficient and the standing wave ratio expressions can be noticed. Some simple methods for leak detection an location are discussed, employing both the static and the dynamic regime. For testing the results, an experimental and an equivalent simulation model are compared. Generally, a good concordance between the theoretical and the simulated results is remarked. Also, this paper proposes a dynamic approach for modeling the noise and vibration generated by the fluid flow, with the aim of testing the acoustic techniques, based on time delay estimation.

REFERENCES