

ECG Signal Acquisition and Analysis for Telemonitoring

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Abstract— The goal of this article is to present an algorithm for QRS complex detection in an electrocardiogram (ECG) signal, realized in Matlab software. The algorithm was tested on real ECG signals acquired with a commercial monitoring system Alive Heart Monitor [1] and also for reference on signals from MIT-BIH online ECG signal database [2]. The goal of our research is to import signals from the monitoring system to a PDA or a Smart Phone via wireless network for signal processing and use in telehealthcare and telemonitoring. Therefore, we examine the concept of wireless acquisition and digital signal processing of ECG (electrocardiogram) signal, which, in addition to traditional medicine is increasingly used in the field of telemedicine as well as in completely non-medical areas, such as sport, entertainment, marketing, etc. Furthermore, we describe the fundamental architecture of an electrocardiograph and describe and overview the basic methods for ECG signal processing.

I. INTRODUCTION

Impact of ECG signal monitoring and analysis systems is increasing rapidly in the present time mainly due to the known problem of an aging population and the increasing incidence of chronic heart disease and sudden heart failure and rapid development of technology. In future, the importance of such systems will only rise as they are penetrating in areas of entertainment, sports, preventive healthcare and telemedicine, where a strong emphasis on active patient participation in the process of treatment is set also by means of self-monitoring and especially tele-monitoring of ECG signal.

In order to present the concept of computerized signal analysis we designed an algorithm for QRS detection based on known methods in Matlab environment. Basic steps of the algorithm are: import of an acquired signal, pass-band filtering of the signal, QRS detection and finally calculation and visualization of the result. The algorithm was tested on real ECG signals, as well as on signals from MIT-BIH online ECG signal database [2]. Computer analysis of ECG signal is divided into two branches - the detection of QRS complexes and their classification. Köhler, Henning and Orglmeister [3] have divided algorithms for detection of QRS complexes or beats into algorithms based on nonlinear filtering and signal transformation [4], algorithms based on wavelet transformation [5], algorithms based on neural networks [6] and algorithms that use additional approaches [7] – [9]. There

is also an approach that exploits the properties of two-dimensional phase portrait, known as delay-coordinate mapping [10]. Detection of QRS complexes is the beginning and foundation of any analysis and application of the signal, and thus represents one of the most important tasks and its reliability determines the reliability of the entire system for ECG signal analysis.

QRS classification is very important in the analysis of heart arrhythmias, where the beats are classified into several groups: normal, atrial, ventricular and supraventricular beats. Beat classification is based on the shape and width of the QRS complex and the length of time intervals between adjacent beats. According to [11] there are two main methods for classifying beats namely, methods based on correlation with templates [12] and methods based on feature extraction [13].

In order to capture ECG signals, we used a commercial system Alive Heart Monitor [1], which covered the wireless transmission of signals using Bluetooth communication standard [14]. System architecture of an ECG signal acquisition system is fairly simple and has few components, but they must be of good quality and effectively linked between each other because of the high sensitivity of the measuring circuit to external disturbances, such as other interfering biological signals, noise, mains voltage, etc. High sensitivity is a result of low voltage amplitudes of ECG signal on the surface of the skin where the signal is acquired and problems are even more pronounced in mobile systems for remote monitoring of ECG signal. Given the characteristics of a system for ECG signal acquisition, an appropriate analog circuit can remove a large part of noise before the digital signal processing, which becomes easier and faster. The most important part of digital signal processing is the detection of QRS complexes, which represents the basis for any further signal processing for purposes based on classification of detected QRS complexes.

II. REMOTE ECG SIGNAL MONITORING SYSTEM

Conceptual architecture of the system for ECG signal acquisition and remote monitoring is shown in Fig. 1: analog signal acquisition with electrodes, signal amplification, signal processing with a band-pass filter and analog to digital signal conversion. According to [15] there are three types of electrodes, which are also used to capture other biosignals, for

example EMG, EEG, etc.: wet, dry and insulated or capacitive electrodes. Wet disposable electrodes are nowadays prevailing in use. These are also used in our test with Alive Heart Monitor.

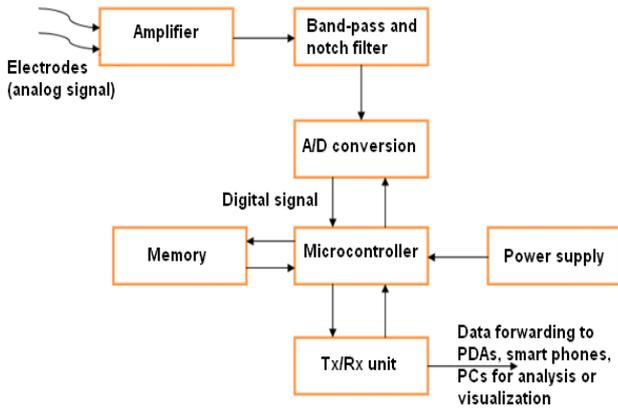


Fig. 1. ECG signal acquisition and remote monitoring system architecture

The signal from electrodes enters the amplifier system. The main issue with the ECG signal which reaches the amplifier is noise, which may have much greater amplitudes than the observed signal. Choosing the right amplifier therefore largely depends on its noise characteristics. For example, common mode interference can be partially abolished by the instrumentation amplifier. To further eliminate interference, we use filters, which are set according to our understanding of the useful signal and interference. Since most of the useful signal is between frequencies 0.05 Hz and 100 Hz [16], the cutoff frequencies of band-pass filter are set accordingly. Both, passive and active filters can be used in this case. Amplifying and filtering stage is followed by sampling and A/D conversion, to prepare the signal for digital processing. Theoretically sufficient is a sampling frequency of 200 Hz. However, in the event that we apply the analysis of ventricular late potentials and heart blocks, we must sample with a minimum of 1000 Hz, since the spectrum of these disorders may extend all the way to 500 Hz. Sampling frequency is therefore chosen depending on the complexity of the ECG signal analysis. For remote signal monitoring the system must be complemented with system memory, microcontroller and transceiver unit using one of wireless communication standards such as Bluetooth [14], ZigBee [17], RFID [18] as well as WiFi [19]. This part of the system transmits the collected data to a remote unit, which both stores the received data, or processes and passes it on.

Systems for ECG signal acquisition differ depending on whether the transmitter-receiver unit supports wireless transmission or if signal processing is possible and especially on the number of electrocardiographic leads that can be monitored at the same time, which is closely related to the number of electrodes used. For each lead two electrodes are required. There are 12 different electrocardiographic leads [20]. In classical medicine, where accurate diagnosis of cardiac disorders and diseases is emphasized, usually all leads are simultaneously monitored, which is called a 12-lead

electrocardiogram and provides a complete overview of cardiac function. On the other hand, one lead monitoring is sufficient for mobile devices and applications for purposes in other areas (sport, telemedicine ...), where the emphasis is on determining heart rate and the most serious malfunction of the heart (arrhythmia, cardiac arrest ...). It offers enough information to effectively monitor the heart rate, which is the principal indicator of cardiac function.

However, any data acquisition is meaningless without a proper analysis and interpretation of its results. Since we already decided to use a low energy consumption commercial device for ECG signal acquisition, we wanted to maintain a similar trend in the choice of an algorithm for computer analysis of the signal.

III. COMPUTER ANALYSIS OF ECG SIGNAL

A. Algorithm Selection and Description

There are a number of methods proposed [3] – [10]. Our selection of algorithm was motivated by [21], which aligns with section II. It is a nonlinear dynamics method, which derives from the chaos theory algorithm and uses the phase portrait reconstruction for its operation. The method was firstly proposed by Shaw [22] and Takens [23], who showed that it is possible to reconstruct a phase portrait, equivalent to a given dynamic system by successive measurements with a single variable of the system. It was demonstrated in embedded systems in [10] and [24]. Following [24], algorithm satisfies the requirements, such as small code size, accurate detection of QRS complexes even in a noisy environment and feasibility for small microcontrollers. Beside [10], algorithm was used for ECG analysis also in [25] and [26].

The delay-coordinate mapping is used for interpretation and visualization of the ECG signal. The main advantage of this method is simultaneous presentation of several specific characteristics of the signal, among other things, P and T waves and of course QRS complexes [20]. All of these three major peaks of the signal cause reconstruction specific trajectories and shapes in the phase portrait. These shapes are called polygons and based on them, we can distinguish between the before mentioned peaks. In our case we have reconstructed the phase portrait in two-dimensional space, where three different polygons arise according to the different peaks in the signal. For the purpose of determining the QRS complexes, we calculated the areas of individual polygons, which determined the detection function.

According to [21], the simplest method to reconstruct the phase portrait is the Takens method of delays [23], which can determine the attractor of a dynamic system in k -dimensional state space from a one-dimensional time series, which represents the measured signal. For example, we have the ECG signal, which is described by a one-dimensional time series of measured scalar values $\{y(t)\}$. From this signal we can construct a n -dimensional signal $\mathbf{Y}(t)$:

$$\mathbf{Y}(t) = \begin{pmatrix} y(t) \\ y(t+\tau) \\ \vdots \\ y[t+\tau(n-1)] \end{pmatrix}, \quad (1)$$

where τ is a time delay and n is a mapping dimension of the reconstruction space, in our case, $n = 2$. The result of the reconstruction process is the attractor presented in n -dimensional space, in our case the phase portrait in two-dimensional space ($y(t), y(t + \tau)$).

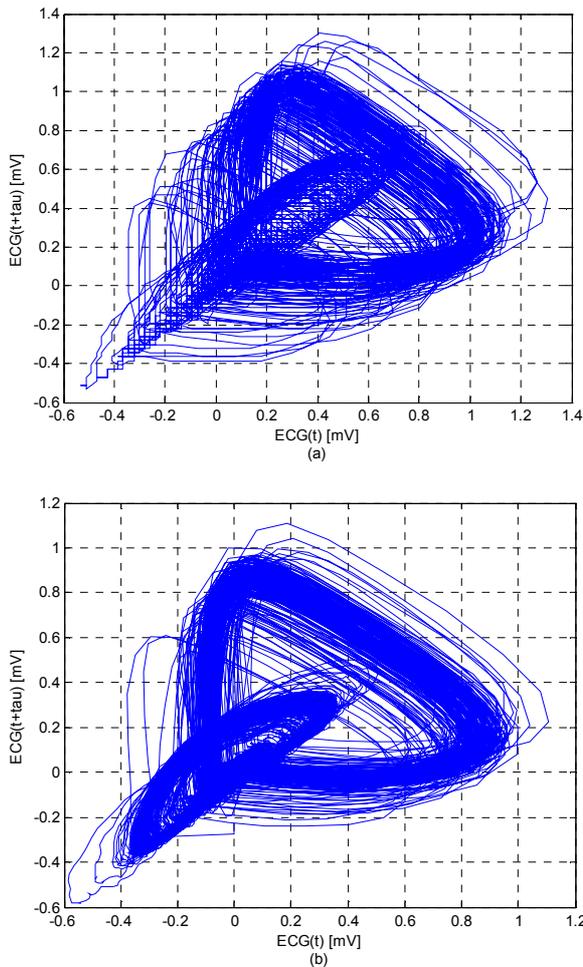


Fig. 2. Phase portrait: of a non-filtered ECG signal (a) and of a band-pass (0.5 Hz, 100Hz) filtered ECG signal (b)

Since the space dimension is determined, the phase portrait depends entirely on the choice of delay. If τ is too small, then the value of $y(t + \tau)$ is near $y(t)$ and the resulting phase portrait is concentrated around the diagonal axis of the space ($y = x$). On the contrary, if τ is too big, geometrical deformation of phase portrait occurs. In our case we determined the right τ experimentally and the result matched the one from [10].

Despite the before mentioned robustness, the method is somewhat sensitive to the presence of high-frequency noise in the signal, caused by muscle contraction, poor electrode contact with the skin, etc. [16], which cause distorted

trajectories and shapes of polygons in a phase portrait. The sensitivity is lower for low frequency noise due to breathing, base level drifting, etc., which leads to moving of the polygons across the phase portrait. However, this can become very disturbing with high amplitudes of noise. Nevertheless, a large proportion of interference and noise can be removed with previous band-pass filtering of the signal. Fig. 2(a) shows the impact of high- and low-frequency noise in the case of the phase portrait of one-channel ECG signal. The largest polygon is difficult to determine because it is covered by a number of trajectories, where one trajectory corresponds to a single cardiac cycle. Among low frequency interference, baseline drift is most noticeable, because it stretches the phase portrait diagonally in the space, along the $y = x$ axis. In addition, high-frequency noise is causing overlapping of polygons and their distorted form and trajectories. On the other hand, Fig. 2(b) shows the phase portrait of the same signal, which was pre-processed with a band-pass filter. We can see significant improvement in the picture as the trajectories of most polygons converge and form clear shapes of polygons. Also, there is less overlapping of polygons, which are therefore more clearly visible.

B. Algorithm Implementation

This article presents a concept of the algorithm for QRS detection on real ECG sample signals acquired by a commercial system Alive Heart Monitor [1], which has basically the same architecture, as shown in Fig. 1. Main advantages of this system are its mobility, small size, long battery life and application of Bluetooth communication standard. Its weakness is the lack of a display for visualization of the acquired signal. In order to visualize the captured signal and the results of signal processing we have to use a smart phone or a computer that support the Bluetooth standard for data transfer. In our case we transferred the acquired signals to a personal computer, where we used Matlab software for visualization and signal processing. The processing algorithm is based on the use of phase portraits properties. We also tested it on some signals from MIT-BIH database [2].

The algorithm was designed, implemented and tested in Matlab environment utilizing Biosig toolbox [27]. The algorithm consists of 6 steps: import of acquired signal, pass-band filtering of imported signal, determination of time-delayed signal and phase portrait, calculation of the detection function values, QRS detection and result calculation and visualization. Below each step is described.

Initially, the acquired signal is imported into workspace using `sload` function from Biosig Toolbox for data conversion to EDF format. Signal acquired with the Alive Heart Monitor, is shown in Fig. 3 (a), with visible baseline drift. By testing, we found that this disorder is result of movement and breathing of the observed person, which causes low frequency components in the signal. We were able to overcome the problem with band-pass filtering according to section II.; the result is shown in Fig. 3 (b), which is realized in step two.

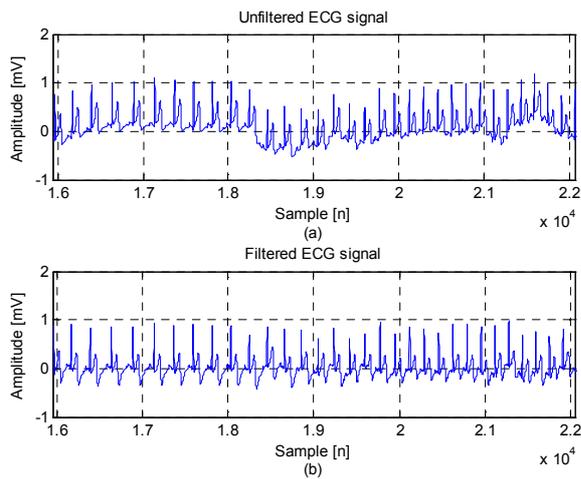


Fig. 3. Section of an acquired, unfiltered ECG signal (a) and the same signal section after band-pass filtering (b)

Unfortunately, with filtering part of the signal information is lost, but on the other hand we emphasized QRS complexes, which represent the most significant and highest peak of the ECG signal, and are most important for our analysis. It is best to set the lower cut-off frequency just above 0 Hz and the upper cut-off frequency to half of the sampling frequency, which was 300 Hz in our case.

In step three, the time-delayed signal was firstly determined according to (1) with the mapping dimension 2. Then a 2-D phase portrait was constructed depending on the given time delay τ (Fig. 4). Coordinates of each data point are determined as $x[nT] = ECG[nT]$ and $y[nT] = ECG[(n-\tau)T]$, where n denotes the sample index.

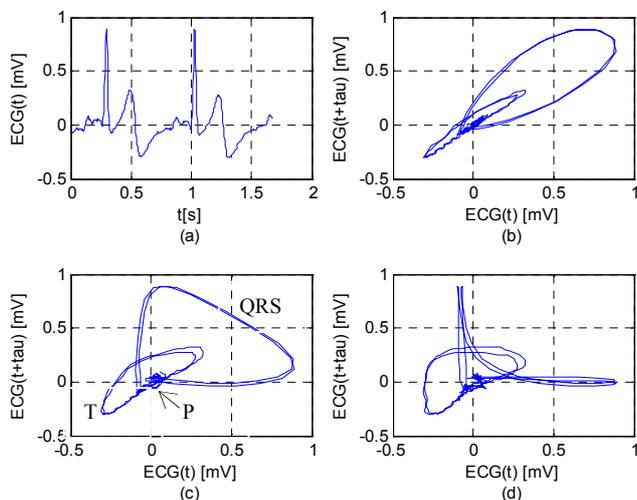


Fig. 4. Phase portraits of an ECG signal (a) at different time delays: 5 ms (b), 20 ms (c) and 40 ms (d)

Each wave-like change in the input signal reflects in the creation of a polygon in phase portrait. The size of the polygon matches the size of the change and its shape depends on the selected time delay. Fig. 4 shows two beats from a filtered ECG signal and the corresponding phase portraits with

different time delays. Individual polygons of the phase portrait have maximum area size in Fig. 4 (c). The largest polygon in Fig. 4 (c) corresponds to the QRS complex, the medium polygon to T wave and the minimum area size polygon corresponds to the P wave. For a correct QRS detection it is essential to choose a suitable time delay because of its impact on the polygon shape. If the delay is too small, the polygons take the form of a narrow ellipse (Fig. 4 (b)) and are not clearly easily visible. Consequently, it is also difficult to correctly calculate their surface area and the detection function values. Similar problems occur, if the time delay is too long. In this case, polygons overlap and cross (Fig. 4 (d)), which results in worse detection rate. The optimal delay setting 20 ms (Fig. 4 (c)) was determined experimentally. The results are in line with [17]. The detection algorithm returns good results even with delays between 15 ms and 25ms.

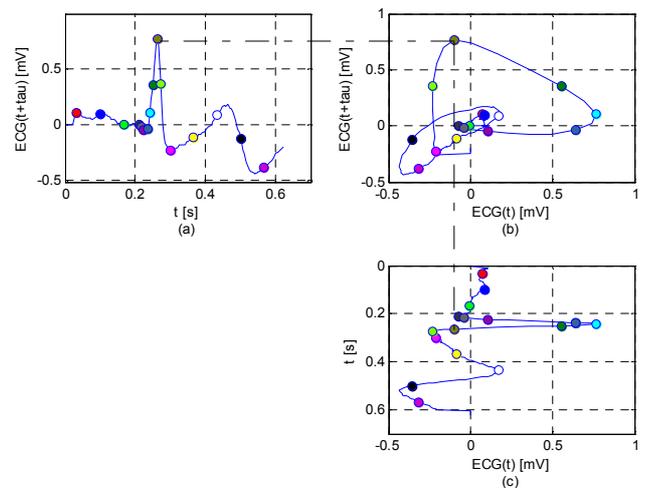


Fig. 5. Construction of phase portrait (b) with values of ECG signal on x-axis (c) and values of the delayed same ECG signal on y-axis (a)

Fig. 5 visualizes the construction of the phase portrait from Fig. 4 (c) using sixteen correlated points in every figure. Correlated points are marked with the same color where one set of points are connected with a dash-dot line. Fig. 5 (a) and Fig. 5 (c) are positioned respectively to the axis they represent in the phase portrait in Fig. 5 (b).

Phase portrait with its polygons, corresponding to ECG signal peaks, represents the basis for the detection function, together with QRS complex detection. To illustrate, elements of the detection function correspond to values of areas of each individual phase portrait polygon, which are calculated by using plane geometry equation for a planar non-self-intersecting polygon area calculation

$$S = \frac{1}{2} \left(\text{abs} \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} + \text{abs} \begin{pmatrix} x_2 & x_3 \\ y_2 & y_3 \end{pmatrix} + \dots + \text{abs} \begin{pmatrix} x_{n-1} & x_n \\ y_{n-1} & y_n \end{pmatrix} \right) \quad (2).$$

Since polygon orientation is not important, we only considered the absolute value of the calculated area. More significantly, parameter n in equation (2) denotes the number

of samples, which comprise one area size. Optimal number of data points, used for forming one area depends very much on the sampling frequency. Algorithm, we implemented, calculated size of areas, comprised of ten samples at sampling frequency 300 Hz as proposed in [24]. To summarize, areas, calculated from (2) represent elements of detection function (Fig. 6) and since QRS complexes formed the largest polygons with the largest surface area they also represented the highest peaks in the detection function.

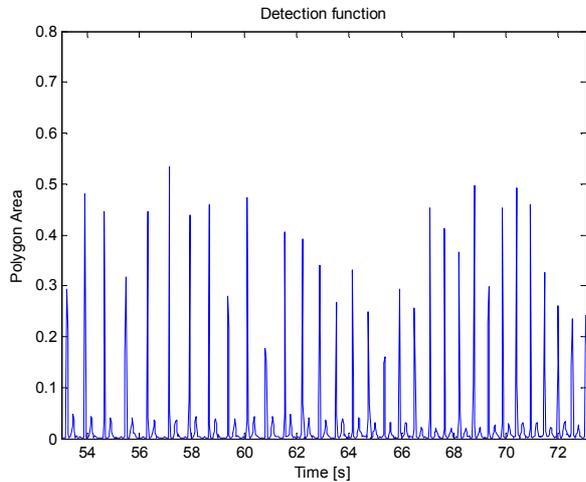


Fig. 6. Detection function values of algorithm for QRS complexes

This was followed by QRS complex detection in the fifth step of the algorithm. For determination of the QRS complexes from the detection function we used a threshold function, which returned amplitude and location of the peaks as a result. Finally, in the last, sixth step the detected QRS complexes were counted, and from their locations in the signal, we were able to calculate the values of individual beats and also the average value of all beats in the observed interval.

C. Mathematical description of phase space

Reconstruction of phase space is based on the analysis of dynamic systems by delay maps, where the signal is represented by the coordinates of a point $y[k]$ in time space.

From a set of scalar numbers $y[k]$, we formed a set of pairs of points $\{(y[k], y[k + \tau]), k = 1, \dots, N\}$, where $y[k]$ denotes the k -th point of the signal and $y[k + \tau]$ is the $(k + \tau)$ -th point of the time series considered and N is the embedding dimension, which is the number of coordinates of the phase space plot. Constructed vector of time series is topologically equivalent to the original dynamics. The value τ can be chosen by user and depends from the task of analysis. One way to choose τ is to take it as the time it takes the autocorrelation function of the data to decay to $1/e$ [28]. Another method is to take the first minimum in the graph of average mutual information, which appears to be better since it considers the nonlinear structure in the signal. The main question is how properties of reconstructed phase space depend on the value τ . From the considered examples on Fig. 4 we can see that the shape of the trajectory is sensitive to changes in delay values that can be

very convenient and informative in analyzing the various characteristics of the signal. However, the unpredictable behavior of the signal until the randomness can lead to loss and distortion of data using the method of delays. Moreover, when τ is more than 40 ms, it is very difficult to determine the QRS complex by using only areas. This is explained by the fact that portrait is deformed in such way that the squares of loops are deformed with increasing delay and complicate the task of analysis. For simplification purposes we perform normalization so that all values were within the range (0,1)

$$y^*[k] = \frac{y[k] - \min y[k]}{\max[k] - \min[k]}, \forall k = 1, \dots, N$$

$$y^{**}[k] = \frac{y[k + \tau] - \min y[k + \tau]}{\max y[k + \tau] - \min y[k + \tau]}, \forall k = 1, \dots, N$$

As a result, we obtain a new set of points $A = \{(y^*[k], y^{**}[k]), k = 1, \dots, N\}$ belonging to the trajectory of the observed signal in two-dimensional normalized space $y^* - y^{**}$. Now $y^*[k] \in [0,1]$ and $y^{**}[k] \in [0,1]$. Plotting the normalized vector from Fig. 7 (a) discloses a phase space portrait on Fig. 7 (b). All the points are doubled in the 2-dimensional phase space. Points of R peaks from both non-delayed and delayed signal are marked with red circles and stars on Fig. 7 (c) and matching of those points is clearly shown in Fig. 7 (d), which represents the signal reconstruction from the phase space.

Correspondence between spaces can be used for further analysis of signal. For example, in phase space can be calculated ratios between points S and T (or R) for characteristic of samples by using simple geometrical definition of Euclidian distance.

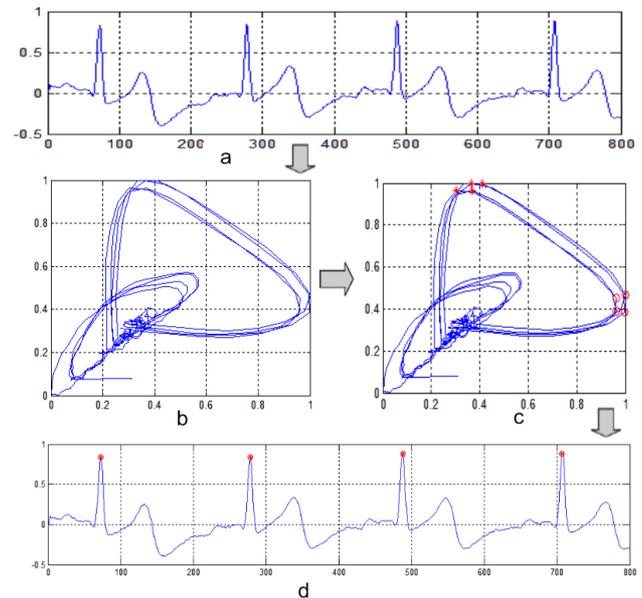


Fig. 7. The sequence of stages of ECG processing - filtered ECG (a); phase trajectory with the normalized coordinates (b), (c). Red circles and stars indicate R peaks; filtered signal with reconstructed R peaks from phase space (d)

IV. RESULTS

The phase portrait algorithm for QRS detection proved its robustness as proposed in [24]. Results are comparable with the Alive Heart Monitor system. The differences were minimal in case base line drift was efficiently eliminated. The presented results correspond to signals acquired from healthy subjects. The algorithm was tested also on a selection of signals from MIT-BIH databases (Arrhythmia Database, Long-Term ECG Database and Normal Sinus Rhythm Database). Slight errors occurred in results of the algorithm, especially with signals containing arrhythmias and abnormal beats, where false QRS complexes were detected. However, we were able to solve these problems with abnormal beats with minor adjustments of the algorithm.

V. CONCLUSION

In the article we have presented an algorithm for QRS complex detection in an electrocardiogram signal, implemented in Matlab environment. To represent the performance of the algorithm, we tested it on real ECG signals, acquired with a commercial monitoring system Alive Heart Monitor [1], as well as on signals from MIT-BIH online ECG database [2] for reference purpose. Results of the algorithm are very promising, since the algorithm was fast and exact in QRS detection.

The goal of our research is to import ECG signals from the monitoring system to a PC, PDA, mobile or smart phone via wireless network for signal processing. This is a foundation for development and penetration of a wide range of services for personal and professional use of ECG signal in the fields of medicine and telemedicine and especially telehealthcare and telemonitoring. Therefore, a concept of wireless acquisition and digital signal processing of ECG signal is presented in the article, comprising of the Alive Heart Monitor for signal acquisition and the selected algorithm for signal processing.

The future of ECG signal acquisition and processing for services of telemonitoring is therefore promising, given the need for such systems are increasingly penetrating into modern life, which puts forward the importance of healthy lifestyles, keeping healthy and use of custom medical and non-medical services based on the user's biological signals.

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