Improved Modeling of Switched Reluctance Motor Including Mutual and Saturation Effects

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Abstract- An advanced nonlinear model of four phase switched reluctance motor (SRM) has been developed. The proposed model takes account of the existence of saturation and mutual inductances. A magnetic data based on the finite element method (FEM) has been generated to check the case of single and doubly energized phases. In this data, the self and mutual inductances and the electromagnetic torque have been computed for wide range of current levels and rotor positions. The present predictions are used as look-up tables in order to guess the behavior of the SRM. Finally, as a demonstration, simulation results are presented to validate the proposed model in both single and doubly excited phase’s mode.

NOMENCLATURE

\[ \begin{align*}
\theta & \quad \text{mechanical position} \\
i_k & \quad \text{current in phase k} \\
R & \quad \text{phase resistance} \\
C_{em} & \quad \text{electromagnetic torque} \\
C_t & \quad \text{load torque} \\
L_k & \quad \text{self inductance of phase k} \\
M_{BC} & \quad \text{mutual inductance} \\
D_0 & \quad \text{friction coefficient} \\
\Phi_k & \quad \text{linkage flux of phase k} \\
J_0 & \quad \text{rotor inertia} \\
\Omega & \quad \text{rotor speed} \\
H & \quad \text{magnetic field intensity} \\
J & \quad \text{total DC current density} \\
B & \quad \text{magnetic flux density} \\
\mu_0 & \quad \text{permeability of free space} \\
\mu_r & \quad \text{permeability of each material} \\
A & \quad \text{magnetic vector potential}
\end{align*} \]

I. INTRODUCTION

Contrary to AC and DC machines, the SRM does not have windings in its rotor. This property makes the design of such machine a simple task. Consequently the use of this machine in industrial applications becomes very attractive. The intrinsic simplicity, robustness and low price of SRM lead to important competition in several variable speed and positioning applications [1]. The simplicity of the motor construction, leading to an interesting manufacturing cost, added to its elevated power density and high efficiency have motivated researchers’ interest [2]. Due to its geometry characterized by a double saliency, the SRM is known as highly nonlinear system. Indeed, the overlapping stator and rotor teeth increase saturation. This effect induces nonlinear magnetic characteristics depending on both rotor position and current level (i.e, flux linkage, inductance and torque). In addition, for two phases excited simultaneously, the mutual inductance is not negligible and also it varies with position and currents. Taking account of its proprieties, these characteristics cannot be exactly described by analytical functions. Consequently, the SRM modeling and analysis is a difficult task [3]. On the other hand, modeling is essential for the description of the dynamic behavior which makes it possible to design adequate control laws. Motivated by this necessary requirement, researchers propose various techniques of modeling. Several authors use analytical approximations, others exploit numerical techniques for the prediction of the unknown parameters. Analytical methods are generally articulated on the approximation of inductance by sinusoidal function, Fourier series or curve fitting. In [4] and [5], self inductance is approximated by a sinusoidal function depending only of the position. This consideration neglects the variation of inductance according to the current which is not the case for the SRM [6]. In the same way, modeling and simulation of linear and nonlinear analytical model of an SRM with single pulse operation has been done in [7]. The 2-D bilinear spline is used in [8] to model the non linear magnetic characteristics in SRM. The Fourier series expression limited to the fourth harmonic and curve fitting method were used in [9] to model the flux linkage according to the inductance data. In spite of all, the main defect of these methods resides in the results precision [10]. The numerical technique is based on finite element method (FEM) analysis, which is recognized for the machine having complex magnetic circuit geometry and non linear properties of the magnetic materials. The FEM is used in [11] and [12] to compute the electromagnetic characterization of an 8/6 SRM. To improve the SRM performances, the FEM can be combined with other techniques. This method, associated with lookup table’s technique, gives good results in nonlinear modelling of the SRM [10]. In above solutions only one excited phase mode is studied, so the mutual inductance is neglected. However, the currents in adjacent machine phases
overlap for a significant portion of the conduction cycle then the mutual inductance cannot be neglected [6].

It is known that the exploitation of this machine for high precision positioning purposes requires an accurate model. Motivated by the over mentioned advantages of the SRM and the reasonable results, obtained when the FEM is associated to lookup table’s technique, the main aim of this work is the development of an advanced nonlinear modeling of an 8/6 SRM. The proposed model includes saturation and mutual effects. It takes into account single and doubly energized phase’s mode.

After a brief description of the considered machine, a full study based on 2-D FEM analysis to acquire the magnetic characteristics, is presented in section III. In section IV, the SRM nonlinear model with two excited phases is elaborated. A simulation results is then offered and discussed, showing the dynamical behaviour of the studied machine in stepping mode operating.

II. THE SRM DESCRIPTION

The motor described in this work is consisted of 8 stator poles and 6 rotary teeth with four phases denoted A, B, C and D. Thus, this actuator is known as a doubly salient machine. Each two diametrically opposite poles constitute one phase. For instance, the winding A1 and A2 are series-connected making so the phase A as it is depicted in Fig. 1.

Whenever one of the four phases is excited, the rotor is aligned with the corresponding stator poles accordingly to the minimum reluctance principle. Considering the rotor position indicated above, if the phases A, B, C and D are energized in turn, the rotor will move in anti-clockwise direction. As a result, the machine can be used either for step by step operation or for driving applications.

III. TORQUE AND INDUCTANCES COMPUTATION

The electromagnetic torque, the self and the mutual inductances of the SRM are nonlinear characteristics which depend on both the currents and the rotor position. For this reason, it is difficult to describe these characteristics by analytical formula. One of the most used techniques in the calculation of such properties is the finite element method (FEM) [13]. It takes into account not only the mutual effect between excited phases but also the saturation phenomenon which can be occurred especially in the doubly salient structure. In the electromagnetic analysis, this method is flexible enough to be undertaken in the problems involving movement such as the SRM and gives results that are well compared against the real behavior.

In the present analysis, the two dimensional finite element software named “Maxwell 2D” is used to determine the static torque characteristics and the inductances profiles of the machine when two phases are excited simultaneously. The characteristics of the analysed machine, summarised in the table below, are introduced with the finite element method to achieve the adequate discritization of such geometry.

| TABLE I |
| GEOMETRY AND TURN INFORMATION FOR THE SRM [14] |
| Outside radius | R_o = 102mm |
| Radius at top of stator pole | R_so = 87.9mm |
| Stator radius at airgap | R_s = 58.5mm |
| Rotor radius at airgap | R_r = 58.2mm |
| Rotor back iron radius | R_ri = 36.4mm |
| Shaft radius | R_s = 18.3mm |
| Stator pole arc | β = 20.1° |
| Rotor pole arc | β = 23.6° |
| Radial airgap | G = 0.3mm |
| Active length | D = 170mm |
| Number of rotor poles | N_r = 6 |
| Turns per pole | N = 70 |
| Rated current | I_n = 16A |

In order to make up these most important characteristics, a virtual work method is used to determine static torque values by computing the magnetic vector potential A over the machine cross section. The equation, that the magnetostatic field solver computes, is derived from Ampere’s law

\[ \nabla \times H = J \]  \hspace{1cm} (1)

with

\[ H = \frac{B}{\mu_0 \mu_r} \]  \hspace{1cm} (2)

and

\[ B = \nabla \times A \]  \hspace{1cm} (3)

Using (2), and substituting (3) into (1), one finds

\[ \nabla \times \left( \frac{1}{\mu_0 \mu_r} \nabla \times A \right) = J \]  \hspace{1cm} (4)
With appropriate boundary conditions and a given current density, the magnetostatic field simulator solves equation (4) using the finite element method. When the magnetic potential vector $A$ is computed, the magnetic flux density $B$ and the magnetic field $H$ can then be found using the previous relationships. Therefore, the torque and the inductances may be deduced from the magnetic flux and total coenergy.

Initially, the dimensions of the studied SRM are used to build a 2D Model. The rotor and the stator are assumed to be made of ferromagnetic material having a non-linear B-H curve. Windings and shaft are made of cooper and cast iron respectively. The airgap region has unit relative permeability. In this 2D model, the source is assigned prescribed values and each area of the physic domain is discretized into a given number of small triangles. The mesh size refinement is so important to enhance good accuracy of the predicted results. Consequently, a large number of triangle elements in the airgap area, is needed.

Fig. 2 shows the magnetic flux distribution of SRM for two energized phases. It is noticed that only these two excited phases are coupled magnetically and their mutual inductance has to be accounted for. As can be seen the magnetic flux path depends on the number of excited phases.

The self and mutual inductances at various current levels are achieved with respect to the different positions. In Fig. 3(a), (b), (c) and (d) are illustrated the self-inductances of each phase considering that only the phase B is excited. For the range of values starting from 3 to 18 A incremented by 3A, it is shown that the self-inductances of the inactive phases (A, C and D) are not affected. Nevertheless, a meaningful variation of self-inductance is pointed out in the phase B which justifies the dependence of the latter on both the current and the position. These conclusions are available for any excited phase.

In Fig. 3(e) is represented the variation of mutual inductance between excited phases accordingly to the position for $I_{c}=3A$ and $I_{B}$ covering the range above-mentioned. Also, the torque is plotted in Fig. 3(f) for the same values of currents in order to use it as a function in the dynamic study in the case of a machine operating with two excited phases. Note that while approaching the aligned position the produced torque decreases significantly. This is caused by the saturation effect, which decreases the coenergy variation [10]. Furthermore, it is seen from the presented results that the magnetic characteristics are strongly nonlinear functions. This deduction can justify the complexity to formulate an accurate model of the studied SRM. Based on the FEM, it is made possible to build a solid magnetic data so that the resulting torque, inductances, phase currents and position will judiciously cover the intended operating region. This data is susceptible to be introduced within the more advanced dynamic model and checking the behavior of the interested machine.

![Fig. 2. Flux distribution during simultaneous excited phases B and C](image)

![Fig. 3(a). Self inductance of phase A](image)

![Fig. 3(b). Self inductance of phase B](image)

![Fig. 3(c). Self inductance of phase C](image)
IV. NON LINEAR SRM MODELING

It is clear from the results given in the previous section that the SRM behavior is inevitably dependent on saturation phenomenon and mutual effect. Indeed, the self inductance of the excited phase is function of not only the rotor position but also the phase current. In addition, the mutual inductance cannot be neglected and also it depends on both position and current of energized phases. For these facts, the main aim of this section is the design of a generalized model of the considered SRM. As only two phases B and C are excited simultaneously, the machine is governed by the following electrical equations:

\[ U_B = R_{i_b} + \frac{d\Phi_B}{dt} \]  \hspace{1cm} (5)

\[ U_C = R_{i_c} + \frac{d\Phi_C}{dt} \]  \hspace{1cm} (6)

\( \Phi_B \) and \( \Phi_C \) are the flux linkages, expressed respectively as follows:

\[ \Phi_B = L_B (i_b, \theta) i_b + M_{BC} (i_b, i_c, \theta) i_c \]  \hspace{1cm} (7)

\[ \Phi_C = L_C (i_c, \theta) i_c + M_{BC} (i_b, i_c, \theta) i_b \]  \hspace{1cm} (8)

Substituting (7) and (8) into (5) and (6) respectively, one may obtain:

\[ U_B = R_{i_b} + \frac{dL_B}{dt} i_b + L_B \frac{di_b}{dt} + M_{BC} \frac{di_c}{dt} + M_{BC} \frac{di_b}{dt} \]  \hspace{1cm} (9)

\[ U_C = R_{i_c} + \frac{dL_C}{dt} i_c + L_C \frac{di_c}{dt} + M_{BC} \frac{di_c}{dt} + M_{BC} \frac{di_b}{dt} \]  \hspace{1cm} (10)

Since \( L_B \) and \( L_C \) are functions of two variables and \( M_{BC} \) is a three variables function, then they should be differentiated as:

\[ \frac{dL_B}{dt} = \frac{\partial L_B}{\partial i_b} \frac{di_b}{dt} + \frac{\partial L_B}{\partial \theta} \frac{d\theta}{dt} \]  \hspace{1cm} (11)

\[ \frac{dL_C}{dt} = \frac{\partial L_C}{\partial i_c} \frac{di_c}{dt} + \frac{\partial L_C}{\partial \theta} \frac{d\theta}{dt} \]  \hspace{1cm} (12)

\[ \frac{dM_{BC}}{dt} = \frac{\partial M_{BC}}{\partial i_b} \frac{di_b}{dt} + \frac{\partial M_{BC}}{\partial i_c} \frac{di_c}{dt} + \frac{\partial M_{BC}}{\partial \theta} \frac{d\theta}{dt} \]  \hspace{1cm} (13)

Finally, the global nonlinear model of the SRM including electrical and mechanical equations is written as:

\[ U_B = R_{i_b} + (L_B (i_b, \theta) + \frac{\partial L_B (i_b, \theta)}{\partial i_b} i_b + \frac{\partial L_B (i_b, \theta)}{\partial \theta} \frac{d\theta}{dt}, \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial i_b} di_c + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial i_c} di_c + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial \theta} \frac{d\theta}{dt}) \frac{di_b}{dt} + (M_{BC} (i_b, i_c, \theta) + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial i_b} i_b + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial i_c} di_c + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial \theta} \frac{d\theta}{dt}) \frac{di_c}{dt} \]  \hspace{1cm} (14)

\[ U_C = R_{i_c} + (L_C (i_c, \theta) + \frac{\partial L_C (i_c, \theta)}{\partial i_c} i_c + \frac{\partial L_C (i_c, \theta)}{\partial \theta} \frac{d\theta}{dt}, \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial i_b} i_b + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial i_c} i_c + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial \theta} \frac{d\theta}{dt}) \frac{di_b}{dt} + (M_{BC} (i_b, i_c, \theta) + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial i_b} i_b + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial i_c} i_c + \frac{\partial M_{BC} (i_b, i_c, \theta)}{\partial \theta} \frac{d\theta}{dt}) \frac{di_c}{dt} \]  \hspace{1cm} (15)

\[ J_n \frac{d\Omega}{dt} = C_m (i_b, i_c, \theta) - C - D_0 \Omega \]  \hspace{1cm} (16)
The difficulty in carrying out this system by numerical simulation consisted in the integration of the nonlinear functions. These functions are mainly the self-mutual inductances and the electromagnetic torque. Most authors proposed analytic solutions to estimate self inductance or torque [10]. But the major defect of this method resides in the results precision especially that the function to be estimated depends on two or three variables. The procedure to apply that permits the nonlinearity of the SRM magnetic characteristic is the use of look-up table with two or three inputs in Matlab/Simulink. This technique estimates with a good precision the inductances and torque according to position, currents in phase B and C. In order to validate this model, the stepping mode is considered where the studied machine is presented in [15]. A study by numerical simulation, considering the case of full stepping mode, is undertaken. For such mode without load, the excitation of only one phase is sufficient. If the rotor is at rest in the position $\theta = 0^\circ$ (see Fig. 1) we know that if we want to step in an anticlockwise direction, the phase B must be energized. Fig. 4 (a) and Fig. 4 (b) show the evolution of current in phase B and the position of the rotor respectively. The power supply sequence B-C-D-A allows the motor to get the first four steps as it is illustrated in Fig. 4 (c). These results are in good agreement with literature [16], [17] and [5].

When load torque is present, however, the rotor will not be able to pull fully into alignment, and a step position error will be unavoidable. [18]. To overcome this drawback, one solution consists in supplying the motor by two phases simultaneously. Indeed, with adequate current levels in phase B and phase C the rotor will reach the equilibrium position corresponding to $15^\circ$ mechanically. The validation of the previously described model with doubly excited phases considering load torque $C_r = 5 \text{ N.m}$, is also succeeded owing to the results shown in Fig. 5.

![Fig. 4. Full step operating without load](image)

![Fig. 4. Full step operating with load](image)
Therefore, the present model is valid in both cases of a machine with single or two excited phases. It constitutes a general and efficient model that may be used in several operating conditions.

V. CONCLUSION

This work has successfully developed nonlinear model of an 8/6 Switched reluctance motor. In this study a detailed analysis using finite element method has been presented. Based on this analysis a data base, describing the different nonlinear characteristics, has been carried out. The determined characteristics were embedded in the dynamical model through look-up tables. The obtained simulation results validate the proposed model for single and double phase modes. Finally, the described model may be very attractive in the control law design for SRM.

REFERENCES